1. Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 inches2 in t minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3 second time interval. A reasonable choice of distribution is P
2. Let X be a random day of the week, coded so that Monday is 1, Tuesday is 2, etc. (so X takes values 1, 2,..., 7, with equal probabilities). Let Y be the next day after X (again represented as an integer between 1 and 7). Do X and Y have the same distribution? What is P(X)

Solution

### Question 1: Raindrops Falling

1. \*\*Choosing the Distribution:\*\*

- The raindrops are falling at an average rate of 20 drops per square inch per minute, which is a typical scenario where events occur independently and at a constant average rate. This suggests that the number of raindrops hitting a particular region over a fixed time period can be modeled using a \*\*Poisson distribution\*\*. The Poisson distribution is often used to model the number of events occurring within a fixed interval of time or space when these events happen independently of each other.

- Let \( A = 5 \) square inches be the area of the region and \( t \) be the time in minutes. The rate of raindrops hitting the region is \( \lambda = 20 \) drops per square inch per minute, so the total rate for the region is:

\[

\lambda\_{total} = \lambda \times A = 20 \times 5 = 100 \text{ drops per minute}

\]

- For a given time interval of \( t \) minutes, the expected number of raindrops is:

\[

\lambda\_t = \lambda\_{total} \times t = 100t \text{ drops}

\]

2. \*\*Computing the Probability:\*\*

- We want to find the probability that the region has no raindrops in a given 3-second time interval.

- First, convert 3 seconds to minutes:

\[

t = \frac{3}{60} = 0.05 \text{ minutes}

\]

- The expected number of raindrops in this time interval is:

\[

\lambda\_{0.05} = 100 \times 0.05 = 5 \text{ drops}

\]

- The number of raindrops in this time period follows a Poisson distribution with parameter \( \lambda = 5 \). The probability of observing 0 raindrops is given by the Poisson probability mass function:

\[

P(X = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-5} \approx 0.0067

\]

- Therefore, the probability that the region has no raindrops in a given 3-second time interval is approximately \*\*0.0067\*\*.

### Question 2: Days of the Week

1. \*\*Distribution of X:\*\*

- \( X \) is the random day of the week coded so that Monday is 1, Tuesday is 2, etc. Since each day of the week is equally likely, \( X \) follows a uniform distribution over the set \( \{1, 2, 3, 4, 5, 6, 7\} \). Thus, the probability mass function of \( X \) is:

\[

P(X = x) = \frac{1}{7} \quad \text{for } x = 1, 2, \dots, 7

\]

2. \*\*Distribution of Y:\*\*

- \( Y \) is the next day after \( X \). Since \( Y \) is also coded as an integer between 1 and 7, it depends on \( X \). Specifically:

\[

Y = \begin{cases}

X + 1 & \text{if } X < 7 \\

1 & \text{if } X = 7

\end{cases}

\]

- Although \( Y \) is determined by \( X \), the distribution of \( Y \) is also uniform over the set \( \{1, 2, 3, 4, 5, 6, 7\} \), because shifting the day by one still gives equal probabilities to each day.

3. \*\*Are X and Y Equally Distributed?\*\*

- Yes, both \( X \) and \( Y \) have the same distribution. They are uniformly distributed over the days of the week, and \( P(Y = y) = \frac{1}{7} \) for \( y = 1, 2, \dots, 7 \), just as with \( X \).

4. \*\*What is P(X)?\*\*

- \( P(X) \) refers to the probability distribution of \( X \), which is uniformly distributed:

\[

P(X = x) = \frac{1}{7} \quad \text{for } x = 1, 2, \dots, 7

\]